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K-Transformation of Genial Labelling of Around Combination of Graphs

Ms. S.Veeralakshmi
Research Scholar
Manonmaniam Sundaranar University
Tirunelveli.

Dr.K.Thirugnanasambandam
Research Supervisor
Manonmaniam Sundaranar University, Tirunelveli
Assistant Professor of Mathematics
Muthurangam Govt Arts and Science College
Otteri, Vellore, Tamilnadu.

Abstract

Let G be a (p; q) graph. Let f : V (G) ! f1; 2; : : : ; kg be a function where k is an integer, 2kjV (G)j. For each edge uv, assign the label jf(u)f(v)j. f is called k-difference cordial labeling of G if jv_f (i) v_f (j)j 1 and je_f (0)e_f(1)j 1 where v_f (x) denotes the number of vertices labelled with x, x 2 f1; 2; : : : kg, e_f (1) and e_f (0) respectively de-note the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate the 3-difference cordial labeling behavior some union of graphs.

Keywords and phrases: Label, Path, Complete graph, Complete bipartite graph, Star.

1 Introduction

Graphs considered here are finite and simple. The union of two graphs G_1 and G_2 is the graph $G_1[G_2$ with $V(G_1[G_2)=V(G_1)[V(G_2)$ and $E(G_1[G_2)=E(G_1)[E(G_2)$. For a graph G, the splitting graph of G,spl(G), is obtained from G by adding for each vertex v of G a new vertex v^0 so that v^0 is adjacent to every vertex that is adjacent to v. Let G_1, G_2 respectively be $(p_1;q_1)$,$(p_2;q_2)$ graphs. The corona of G_1 with G_2, $G_{\ 1}$ G_2 is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. If x = uv is an edge of G and w is not a vertex of of G, then x is subdivided when it is replaced by the lines uw and wv. If every edges of G is subdivided, the resulting graph is the subdivision graph S(G). The graph $P_n + K_1$ is called a fan F_n. The graph $P_n + 2K_1$ is called a double fan DF_n. Cahit [1], introduced the concept of cordial labeling of graphs. Recently Ponraj et al. [4], introduced k-difference cordial labeling of graphs and 3-difference cordial labeling of wheel, helms, flower graph, sunflower graph, lotus inside a circle, closed helm, double wheel, $K_{1;n}$ K_2, P_n $3K_1$, mC_4, spl($K_{1;n}$), DS($B_{n;n}$), C_n K_2, and some more graphs have been studied in [5, 6]. In this paper we investigate the 3-difference cordial labeling behavior of some union of graphs. Terms are not defined here follows from Harary [3].

2 k-Difference Cordial Labeling

Definition 2.1. Let G be a (p; q) graph. Let f:V(G)!f1; 2; : : : ; kg be a map where k is an integer, 2kjV(G)j. For each edge uv, assign the label jf(u) f(v)j. f is called k-difference cordial labeling of G if jv_f (i) v_f (j)j 1 and je_f (0) e_f (1)j 1 where v_f (x) denotes the number of vertices labelled with x, e_f (1) and e_f (0) respectively denote the number of edges labelled with 1 and not labelled with 1. A graph which admits a k-difference cordial labeling is called a k-difference cordial graph.

Theorem 2.2. If G is(p;q) 3-difference cordial graph with p 0 (mod 2) and q 0 (mod 3), then G[G also 3-difference cordial.

Proof. Let f be a 3-difference cordial labeling of G. Then v_f (1) = v_f (2) = v_f (3) = $\frac{p}{3}$ and e_f (0) = e_f(1)= $\frac{q}{2}$. Let h be a map from V(G[G)!f1;2;3g defined by h(u) = f(u) for all 2V(G[G). Clearly v_h(1)=v_h(2)=v_h(3)= $\frac{2p}{3}$ and e_h(0)=e_h(1)=q. Therefore, h is a 3-difference cordial labeling of G[G].

First we investigate the 3-difference cordial labeling behavior of union of graphs with the star.

Theorem 2.3. P_n [$K_{1;n}$ is 3-difference cordial.

Proof. Let P_n be the $v_1 v_2$: : : v_n. Note that P_n [$K_{1;n}$ has 2n+1 vertices and 2n-1 edges.

Case 1. n 0 (mod 3).

Assign the labels 1,3,2 to the first three vertices of the path v_1, v_2, v_3 respectively. Then we assign the labels 1; 3; 2 to the next three vertices of the path v_4, v_5, v_6 respectively. Continuing in this way, we assign the next three vertices and so on. Next we move to the graph $K_{1;n}$. First we assign the label 1 to the vertices u_i (1 i $\frac{n}{3}$). Next we assign the label 2 to the vertices $u_{\frac{n}{3}+i}$ (1 i $\frac{n}{3}$). Then we assign the label 3 to the vertices $u_{\frac{2n}{3}+i}$ (1 i $\frac{n}{3}$). Finally we assign the label 1 t the central vertex u.

Case 2. n 1 (mod 3).

Assign the labels u_i, v_i, u (1 i n 1) as in case 1. Then assign the labels 1,2 to the vertices v_n and u_n respectively.

Case 3. n 2 (mod 3).

As in case 2, assign the labels to the vertices u_i, v_i, u (1 i n 1). Finally assign the labels 3,3 to the vertices u_n and v_n respectively. The vertex and edge condition are given in table

1 and 2 respectively.

Nature of n	e_f (0)	e_f (1)
n 0; 2 (mod 3)	n	n 1
n 1 (mod 3)	n 1	n

Table 1.

values of n	v_f (1)	v_f (2)	v_f (3)
n 0 (mod 3)	$\frac{2n+3}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
n 1 (mod 3)	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
n 2 (mod 3)	$\frac{2n+2}{3}$	$\frac{2n\ 1}{3}$	$\frac{2n+2}{3}$

□

Next investigation is union of star with $K_{2;n}$.

Theorem 2.4. $K_{1;n}$ [$K_{2;n}$ is 3-difference cordial.

Proof. Let V ($K_{2;n}$) = fv; w; v_i:1 ing and E($K_{2;n}$)=fvv_i; wv_i:1ing.
Case 1. n 0 (mod 3).
Subcase 1a. n 0 (mod 6).

First we consider the graph $K_{1;n}$. Assign the labels 1,1,2 to the first three vertices u_1, u_2, u_3 respectively. Then assign the labels 2,2,1 to the next three vertices u_4; u_5; u_6 respectively. Next we assign the labels 1,1,2 to the next three vertices u_7; u_8; u_9 respectively and and assign the labels 2,2,1 to the next three vertices u_{10}; u_{11}; u_{12} respectively. Continuing this way, we assign the next three vertices and so on. Clearly in this process, the last vertex u_n received the label 2 or 1. Finally we assign the 1 to the vertex u. Now we move to the graph $K_{2;n}$. Assign the labels 3,3,2,3,3,1 to the first six vertices v_1; v_2; v_3; v_4; v_5; v_6 respectively. Then we assign the labels 3,3,2,3,3,1 to the next six vertices v_7; v_8; v_9; v_{10}; v_{11}; v_{12} respectively. Proceeding like this, we assign the labels to the next six vertices and so on. Clearly the last vertex v_n received the label

1. Finally, we assign the labels 2,3 to the vertices v and w respectively. Subcase 1b. n 3 (mod 6).

Assign the label to the vertices u, v, w, u_i 1 i n 3 and v_i 1 i n 3 as in subcase 1a. Finally assign the labels 1,1,2 to the vertices $u_{n\ 2}$, $u_{n\ 1}$, u_n respectively and 3,3,2 to the vertices $v_{n\ 2}$, $v_{n\ 1}$, v_n respectively.

Case 2. n 1 (mod 3).

Subcase 2a. n 4 (mod 6).

Fix the labels 1,1,2,2 to the vertices u_1; u_2; u_3; u_4 respectively. Then we assign the labels 2,2,1,1,1,2 to the next six vertices u_5; u_6; : : : ; u_{10} respectively. Now assign the labels 2,2,1,1,1,2 to the next six vertices u_{11}; u_{12}; : : : ; u_{15} respectively. Continuing in this way, we assign the next six vertices and so on. In this process, the last vertex u_n received the label 2. Next we assign the label 1 to the vertex u. Now our attention move to the vertices of the graph $K_{2;n}$. Fix the label 1 to the vertex v_1. Then assign the labels 3,3,2,3,3,1 to the next six vertices v_2; v_3; : : : ; v_7 respectively. Proceeding like this, we asign the next six vertices and so on. Clearly, in this process the vertex $v_{n\ 3}$ received the label 1. Next we assign the labels 3,3,2 respectively to the vertices $v_{n\ 2}$, $v_{n\ 1}$, v_n. Finally we assign the labels 2,3 to the vertices v and w respectively.

Subcase 2b. n 1 (mod 6).

Assign the label to the vertices u, v, w, u_i 1 i n 3 and v_i 1 i n 3 as in subcase 2a. Finally assign the labels 2,2,1 to the vertices $u_{n\ 2}$, $u_{n\ 1}$, u_n respectively and 3,3,2 to the vertices $v_{n\ 2}$, $v_{n\ 1}$, v_n respectively.

Case 3. n 2 (mod 3).
Subcase 3a. n 2 (mod 6).

Fix the labels 1,2 to the vertices u_1, u_2 respectively. Then we assign the labels 1,1,2,2,2,1 to the next six vertices u_3; u_4; : : : ; u_8 respectively. Now we assign the labels 1,1,2,2,2,1 to the next six vertices u_9; u_{10}; : : : ; u_{14} respectively. Continuing this process until we reach the last vertex u_n. In this pattern, the last vertex u_n labeled by the integer 1. Then we assign the label 1 to the vertex u. Next we move to the graph $K_{2;n}$. Fix the labels 1,3 to the vertices v_1, v_2 respectively. Then we assign the labels 3,3,2,3,3,1 to the next six vertices v_3; v_4; : : : v_8 respectively. Next we assign the labels 3,3,2,3,3,1 to the next six vertices v_9; v_{10}; : : : v_{14} respectively. Continuing in this way, we assign the next six vertices and so on. Finally we assign the labels 2,3 to the vertices v,w respectively. The vertex and edge condition are given in table 3 and 4.

Nature of n	$e_f(0)$	$e_f(1)$
n 0; 2; 4 (mod 6)	$\frac{3n}{2}$	$\frac{3n}{2}$
n (1 mod 6)	$\frac{3n\ 1}{2}$	$\frac{3n+1}{2}$
n 3 ;(5 mod 6)	$\frac{3n+1}{2}$	$\frac{3n\ 1}{2}$

Table 3.

Nature of n	v_f (1)	v_f (2)	v_f (3)
n 0 (mod 3)	$\frac{2n+3}{3}$	$\frac{2n+3}{3}$	$\frac{2n+3}{3}$
n 1 (mod 3)	$\frac{2n+4}{3}$	$\frac{2n+4}{3}$	$\frac{2n+1}{3}$
n 2 (mod 3)	$\frac{2n+5}{3}$	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$

Table 4.

We now investigate union of star with subdivision of star.

Theorem 2.5. $K_{1;n}$ [$S(K_{1;n})$ is 3-difference cordial.

Proof. Let V $(S(K_{1;n}))$ = fv; v_i; w_i :1 ing and $E(S(K_{1;n}))$ = fvv_i; $v_i w_i$:1ing.

Case 1. n is even.
Case 2. n is odd.
Assign the label 1 to the vertex u. Then assign the integer 3 to the vertex u_1; u_2; : : : u_{n+1} .

Then assign the label 2 to the remaining vertices of the star $K_{1;n}$. Then we move to the graph $S(K_{1;n})$. Now we assign the label 2 to the vertex v. Then we assign the label 2 to the vertices

Next is union of two stars.

Theorem 2.6. If n0; 1 (mod 3), then $K_{1;n}$ [$K_{1;n}$ is 3-difference cordial.

Proof. Let u; v be the central vertex of the first and second star respectively. Let u_i (1 i n) and v_i (1 i n) be the pendent vertices of first and second copies of the star $K_{1;n}$.
Case 1. n 0 (mod 3).
Assign the label 1 to the vertices u_i, v_i (1i$\frac{n}{3}$) and assign the label 2 respectively. The edge condition is e_f (0) = e_f (1) = 1 and the vertex condition is given in table 6.

values of n	v_f (1)	v_f (2)	v_f (3)
n 0 (mod 3)	$\frac{2n+3}{3}$	$\frac{2n+3}{3}$	$\frac{2n}{3}$
n 1 (mod 3)	$\frac{2n+4}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$

Table 6.

Next investigation is about union of graphs with splitting graph of the star.

Theorem 2.7. spl($K_{1;n}$) [$K_{1;n}$ is 3-difference cordial.

□

Proof. Let V (spl($K_{1;n}$))=fv;w;v_i;w_i:1 ing and E(spl($K_{1;n}$)=fvv_i;vw_i;ww_i :ng. Note that spl($K_{1;n}$) [$K_{1;n}$ has 3n + 3 vertices and 4n edges. Assign the labels 1; 2; 3 to the vertices u; v; w respectively. We now assign the label 3 to u_i (1 i n), assign the label 1 to the vertices v_i (1 i n). Finally assign the label 2 to the vertices w_i (1 i n). It is easy to verify that e_f (0) = e_f (1) = 2n and v_f (1) = v_f (2) = v_f (3) = n + 1. Hence f is a 3-difference cordial labeling.

Now our attention is move to union of graphs with splitting graph of the star.

Theorem 2.8. $K_{3;n}$ [spl($K_{1;n}$) is 3-difference cordial.

Proof. Let V ($K_{3;n}$) = fu; v; w; u_i : 1ing and E($K_{3;n}$) = fuu_i; vu_i; wu_i : 1 i ng. Let V (spl($K_{1;n}$)) = fx; y; x_i; y_i : 1 ing and E(spl($K_{1;n}$)) = fxx_i; xy_i; yy_i : 1 i ng. Clearly $K_{3;n}$ [spl($K_{1;n}$) has 3n+5 vertices and 6n edges. Define a map f : V (G) ! f1; 2; 3g by f(u) = 1, f(v) = 2, f(w) = 3, f(x) = 2, f(y) = 1,

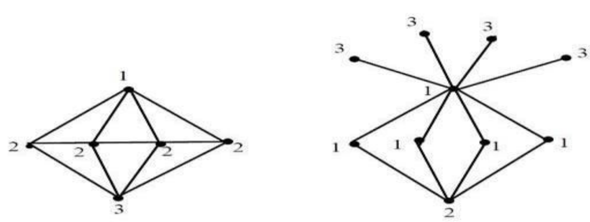
$$f(u_i) = 1; \ 1 \ i \ n$$
$$f(x_i) = 3; \ 1 \ i \ n$$
$$f(y_i) = 2; \ 1 \ i \ n$$

Clearly e_f (0) = e_f (1) = 3n and v_f (1) = v_f (2) = n + 2 and v_f (3) = n + 1. Hence f is 3-difference cordial labeling. □

Theorem 2.10. DF_n [spl($K_{1;n}$) is 3-difference cordial.

Proof. Let V (DF_n) = fu; v; u_i : 1 i ng and E(DF_n) = fuu_i; vu_i; $u_i u_{i+1}$: 1 i ng, V (spl($K_{1;n}$)) = fx; y; x_i; y_i : 1 i ng and E(spl($K_{1;n}$)) = fxx_i; xy_i; yy_i : 1 i ng. Assign the label 1 to the vertex u. Then assign the label 2 to all the vertices v_i (1 i n) and assign the label 3 to the vertex v. Now we move move to the graph spl($K_{1;n}$). First we assign the label 1 to the vertex x. Then assign the label 3 to all the vertices x_i (1 i n) and assign the label 1 to all the vertices y_i (1 i n). Finally assign the label 2 to the vertex y. Clearly v_f (1) = n + 2 and v_f (3) = n + 1, e_f (0) = 3n 1 and e_f (1) = 3n. Hence f is a 3-difference cordial labeling. □

Example 2.11. A 3-difference cordial labeling of DF_4 [spl($K_{1;4}$) is displayed in the below figure.



Conclusion

In this paper we investigate the 3-difference cordial labeling behavior some union of graphs. Graphs considered here are finite and simple. The union of two graphs G1 and G2 is the graph G1[G2 with V (G1 [G2) = V (G1)[V (G2) and E (G1 [G2) = E (G1)[E (G2). For each edge uv, assign the label jf(u):f(v)j. f is called k-difference cordial labeling of G if where vf (x) denotes the number of vertices labelled with x, ef (1) and ef (0) respectively denote the number of edges labelled with 1 and not labelled with 1. A graph which admits a k-difference cordial labeling is called a k-difference cordial graph.

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